10 Partial Order Relation

65. Let $\mathbb{N}$ be the set of natural numbers. On $\mathbb{N}$, we define a relation R by:

$$xRy \iff \exists z \in \mathbb{N} : xz = y.$$

Verify that R is a partial order relation.

66. Let $\mathbb{R}$ be the set of real numbers. On $\mathbb{R}$, we define a relation R by:

$$aRb \quad \Leftrightarrow \quad \exists k \in \mathbb{N}_0 : b = 2^k a.$$

(i) Write at least five elements of the relation R.

(ii) Is R a partial order relation?

(iii) Is R a linear order relation?

Provide detailed justifications for all answers.

67. Consider the divisibility relation $|$, defined on the set of positive integers, where $a|b$ is read as "a divides b".

(i) Show that $|$ is a partial order relation on $\mathbb{Z}^+$.

(ii) Show that $|$ is not a linear order relation on $\mathbb{Z}^+$.

68. Let $S = \mathbb{Z}$ be the set of integers. On S, we define a relation R by:

$$aRb \iff \exists r \in \mathbb{N} : b = a^r.$$

(a) Is R a partial order relation?

(b) Is R a linear order relation?

69. Let $\mathbb{Z}$ be the set of integers. On $\mathbb{Z}$, we define a relation R by:

$$xRy \quad \Leftrightarrow \quad \exists n \in \mathbb{N}_0 : x = y + n.$$

Is R a partial order relation? Is R a linear order relation? Provide detailed justifications for your answers.

70. Let B_4 be the set of natural numbers from 0 to 15. We represent these numbers in binary notation: the number $b \in B_4$ is written as $b = b_3b_2b_1b_0$, where each digit b_i is either 0 or 1 (specifically, $b = b_3 2^3 + b_2 2^2 + b_1 2^1 + b_0 2^0$, e.g., $8 = 1000$, $2 = 0010$, $15 = 1111$, etc.). On B_4, we define a relation $\preceq$ by:

$$a \preceq b \iff \forall i \ (a_i \leq b_i).$$

(i) Write at least five elements of the relation $\preceq$.

(ii) Is $\preceq$ a partial order relation?

(iii) Is $\preceq$ a linear order relation?

Provide detailed justifications for all answers.

All above math problems are taken from the following website:
https://osebje.famnit.upr.si/~penjic/teaching.html.
THE READER CAN FIND ALL SOLUTIONS TO THE GIVEN PROBLEMS ON THE SAME PAGE.